Technical Notes

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Drag Reduction Potentials of Turbulence Manipulation in Adverse Pressure Gradient Flows

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Introduction

MOST work in turbulent drag reduction research has been performed up to now in flat-plate boundary layers at zero pressure gradient. In this method it is evident that to minimize the drag, the skin friction must be reduced as much as possible. The problem is more complex in the general case of flow around bodies of finite thickness. Even when restricting the problem to two-dimensional incompressible flows, two contributions to the drag can be distinguished: skin-friction drag and pressure drag. Especially in adverse pressure gradient regions, the contribution of the pressure drag may become large. In this Note the consequences of this fact will be discussed in relation to turbulent drag reduction.

Viscous Drag

The loss of total pressure in the flow at infinity downstream of a body is a measure of the total viscous drag, i.e., the sum of the skin-friction and the pressure drag. More precisely, the total drag is proportional to the viscous shear layer momentum thickness at infinity:

$$C_d = 2\,\theta_\infty / c \tag{1}$$

This suggests that to minimize the drag, the momentum thickness growth must be minimized at each position along the body and in the wake. In fact, it can be proven strictly that this is only true if the local minimum depends solely on local conditions, which is the case within the limitations of the simple analysis to be presented here.

The increase of the momentum thickness is given by the momentum integral equation

$$\frac{d\theta}{dx} = \frac{1}{2} C_f - (2+H)\,\Gamma \tag{2}$$

Besides the skin-friction coefficient C_f and the shear layer shape factor $H = \delta^*/\theta$, the equation contains the pressure gradient parameter $\Gamma = (\theta/U_e)\,(dU_e/dx)$. Numerical evaluation shows that at somewhat large pressure gradients the contribution of the last term of Eq. (2), which is related to the pressure drag, dominates over the first term at the right-hand side, $1/2\ C_f$.

Turbulence Manipulation

There has been extensive research on turbulence manipulation for drag reduction.[1,2] One of the more promising techniques is placing ribbons parallel to the surface in the outer region of the boundary layer. These affect the turbulence properties over a substantial distance downstream of the manipulator. Mainly the turbulence structure in the outer region is affected, as lasting changes in the boundary-layer inner region are difficult to achieve because turbulence production is high there. Since the inner region is not directly affected, the ordinary law of the wall is likely to remain valid. Actually, recent measurements behind ribbons[3] indicate that, except close behind the device trailing edge with a distinct device wake, the velocity profiles fit well to Coles' law-of-the-wall wake formula.[4] The skin-friction values inferred from the velocity profile fits compare well with the skin-friction data obtained from (direct) measurements. This means that the consequent skin-friction law of Coles should also hold. The formula is, however, less convenient for further algebraic operations and is therefore often replaced by the skin-friction law of Ludwieg and Tillmann:

$$C_f = 0.246\, R_\theta^{-0.268}\, 10^{-0.678H} \tag{3}$$

The agreement between both skin-friction laws is generally good.[5] The important result for the present analysis is that a shape factor increase and a skin-friction decrease go together. On the basis of the foregoing it may be assumed that the same relation between skin friction and shape factor holds for boundary layers with outer region manipulators (except close downstream). Actually, such a mutual relation generally should be expected to exist as long as the ordinary law of the wall remains valid, since in that case a skin-friction increase is necessarily associated with larger velocities in the wall region, i.e., a fuller velocity profile and a smaller shape factor, and just the reverse for a skin-friction decrease.

Drag Minimization

It follows from Eq. (2) that to reduce the momentum thickness growth in adverse pressure gradient flows ($\Gamma < 0$), both the skin friction and the shape factor should be small. As skin friction and shape factor are related, it is not directly evident for which conditions the momentum thickness growth is minimum. To investigate this, it will be assumed that Eq. (3) is valid for boundary layers with outer region manipulators, as argued in the preceding section. Substitution of Eq. (3) in Eq. (2) yields

$$\frac{d\theta}{dx} = 0.123\, R_\theta^{-0.268}\, 10^{-0.678H} - (2+H)\,\Gamma \tag{4}$$

If the viscous-inviscid interaction is weak, U_e and dU_e/dx may be regarded as given. Thus, Eq. (4) relates the local momentum thickness growth $d\theta/dx$ to the local shape factor H and the local momentum thickness θ. It is supposed that by suitable turbulence manipulation the shape factor and the corresponding skin friction can be adjusted freely. Ideal manipulators are assumed, which do not produce extra momentum loss. Sample calculations, including a momentum loss for the manipulator, did not result in essentially different trends.[6]

Minimum momentum thickness growth at given position x with given θ is obtained for $\partial/\partial H\,(d\theta/dx) = 0$. This yields for the optimum shape factor ($\Gamma < 0$)

$$H_{\rm opt} = -1.06 - 0.172\,\ln R_\theta - 0.641\,\ln(-\Gamma) \tag{5}$$

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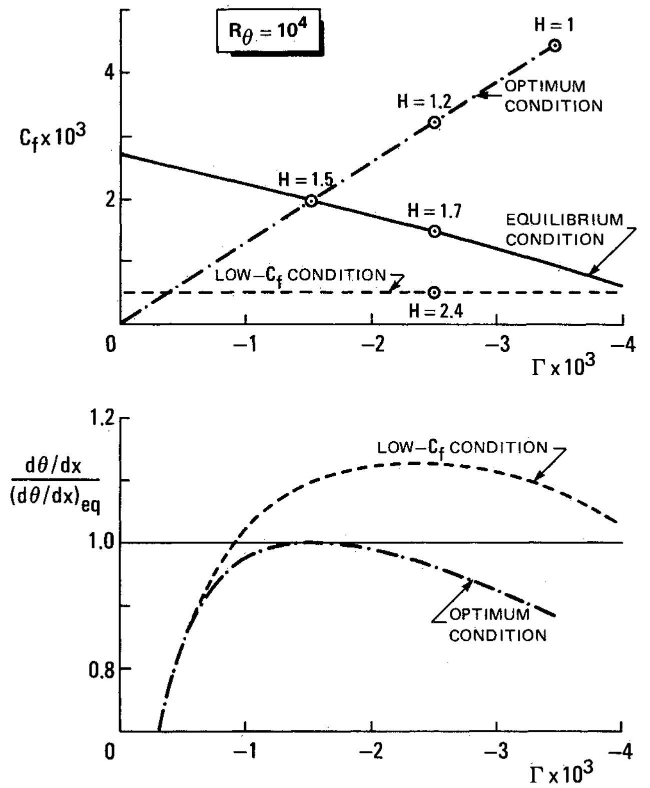

Fig. 1 Boundary layer momentum thickness growth at various conditions.

The corresponding optimum skin friction and minimum momentum thickness growth can also be derived and have been plotted in Fig. 1. The figure also shows the momentum thickness growth, which is obtained when simply aiming at a low skin friction (here $C_f = 0.5 \times 10^{-3}$). The momentum thickness growths are presented relative to the growth in a natural (nonmanipulated) turbulent boundary layer at the same pressure gradient. The skin-friction variation in the natural condition is shown as well. The natural turbulent boundary-layer data have been derived from measurements in nonmanipulated equilibrium turbulent boundary layers.[7]

At zero pressure gradient ($\Gamma = 0$), minimum momentum thickness growth occurs for $C_f = 0$, of course, but with an adverse pressure gradient the minimum appears to be obtained for nonzero skin friction. At larger adverse pressure gradients ($\Gamma < -1.5 \times 10^{-3}$), the skin friction must be increased even above the value in a natural turbulent boundary layer to minimize momentum thickness growth. This occurs at pressure gradients essentially below the gradient leading to flow separation ($\Gamma \approx -4 \times 10^{-3}$ at separation for nonmanipulated boundary layers[8]).

Some shape factor values have been indicated on the curves in Fig. 1. Note that, as essentially $H \geq 1$, the computed optimum curves do not extend beyond $\Gamma \approx -3.5 \times 10^{-3}$. Actually, the accuracy of the underlying skin-friction law, Eq. (3), will probably diminish for $H < 1.2$. Even when taking this into consideration, it is still quite clear from the results that in adverse pressure gradient flows, minimum momentum thickness growth is not obtained when the skin friction is zero, and that fairly high skin frictions may be profitable at larger adverse pressure gradients.

Conclusion

Minimum skin friction does not necessarily mean minimum drag. To minimize drag, account should be taken of both friction and pressure drag. In adverse pressure gradient regions, this means that a low skin friction as well as a small shape factor should be pursued. This restricts the drag reduction potentials, as skin-friction reductions tend to be coupled with shape factor increments. The analysis presented here indicates that at larger adverse pressure gradients this may mean that the skin friction must be increased to reduce the drag.

The general conclusion is that it is not obvious in adverse pressure gradient flows how turbulence must be manipulated to minimize drag. It would be worthwhile to establish theoretically with an optimization method which turbulent shear stress level and distribution would lead at given conditions to minimum drag.

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Applications of Various Coordinate Transformations for Rotating Disk Flow Stability

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Introduction

It has already been established[1] that the instability mechanism of the rotating disk flow is similar to that of other three-dimensional boundary layers. The analytical studies of the rotating disk flow have commonly benefited from existence of the basic flow based on von Karman's[2] similarity solution. Discrepancies between earlier analyses and the experiments[1,3] for the critical Reynolds number were reduced in more recent works[4,5] by including the Coriolis force and streamline curvature. The resulting equations are, however, sixth-order instead of the fourth-order Orr-Sommerfeld equation.

In the present work the linear, temporal stability of the rotating disk flow is investigated by a spectral method based on Chebyshev polynomials with emphasis on the various coordinate transformations.

Formulation

A flat disk of infinite radius rotating about the vertical axis with angular velocity Ω is considered. Applying von Karman's

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